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Rewiev paper

Georgi-Glashow SU(5) - basis for proton decay and neutrino mass problem from SM to GUT

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ABSTRACT

In this article we present the original Georgi-Glashow $SU(5)$ model with gauge and fermion sector derivation in order to show grand unified theory (GUT) through this model, but also to correlate $SU(5)$ and Standard Model (SM). One of the correlations reside in neutrino mass sector where both $SU(5)$ and Standard Model (SM) see neutrinos as massless particles. We will show that gauge sector yields twelve gauge bosons that will mediate proton decay within $SU(5)$. Beside this, Georgi-Glashow model provides a basis for development and extrapolation in terms of reconstructing it to find viable neutrino mass mechanism that we present in our conclusion.

Keywords: Georgi-Glashow; Standard Model; GUT; neutrino mass problem; proton decay

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1. INTRODUCTION

Standard Model is one of the best theories we have today to explain particle content of the world and the interactions among the particles building our Universe. However, this theory does not explain several experimental and theoretical facts that govern physicists to reach for theories beyond Standard Model. One of those theories is $SU(5)$ theory trying to go above unification of Standard Model and reconcile some of the mismatches such as unification scale, neutrino masses and proton decay. Namely, we will tackle the first and original $SU(5)$ Georgi-Glashow [1] model and its advantages but also disadvantages leaving it as a good example, yet not as a solution to Standard Model. Nevertheless, Georgi-Glashow [1] $SU(5)$ inspired many and provided great basis for something more sustainable in particle physics and theories beyond the Standard Model.

2. STANDARD MODEL

Standard Model is the theory that, so far, describes in the most viable manner algorithm of the particle content of the Universe we know of and the forces and fields in the nature. This is the theory that has been forged for decades by many great names in the world of physics such as Chen Ning Yang, Robert Mills, Chien-Shiung Wu, Sheldon Glashow, Abdus Salam, Steven Weinberg, Abraham Pais, Sam Treiman, Philip Warren Anderson, Robert Brout, François Englert, Peter Higgs, Yoichiro Nambu, Jeffrey Goldstone and many more.

2.1. Particle Content

Standard Model is a chiral gauge theory providing all known building blocks of matter and defining three out of four fundamental forces in nature: electromagnetic, weak and strong interactions [2-4]. The mechanism of this theory is based on the gauge group $SU(3) \times SU(2) \times U(1)$, where SU(3) is a gauge group for strong interactions and $SU(2) \times U(1)$ is a gauge group for electroweak interactions. A simplistic categorisation of particle content of the Standard Model would be to introduce two types of particles: fermions that obey Fermi-Dirac statistics and build up the matter fields, and experience the force; and bosons that obey Bose-Einstein statistics with a distinction between gauge bosons of spin 1 that are force carriers, and Higgs boson of spin 0 that is responsible for symmetry breaking of the Standard Model. All the particles of the Standard Model are summarized in simplified manner in Table 1.

Table 1. Standard Model particle content

	FERMIONS			BOSONS	
	families			SPIN 1	SPIN 0
	1st	2nd	3rd		
quarks	u	c	t	gluons	H
	d	s	b	g	
leptons	e	μ	τ	$W^{\pm}$	
	ν_e	ν_μ	ν_τ	Z	

2.2. Lagrangian

Standard Model is a gauge theory, or precisely, it is a quantum field theory. Lagrangian of the Standard Model can be written in the following form:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \tag{1}$$

where individual contributions are [5-6]:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} G^A_{\mu\nu} G^{A\,\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{2}$$

$$\mathcal{L}_{\text{kinetic}} = i\left(\bar{Q}^i_L \partial_\mu \gamma^\mu Q^i_L + \bar{u}^i_R \partial_\mu \gamma^\mu u^i_R + \bar{d}^i_R \partial_\mu \gamma^\mu d^i_R + \bar{l}^i_L \partial_\mu \gamma^\mu l^i_L + \bar{e}^i_R \partial_\mu \gamma^\mu e^i_R\right) \tag{3}$$

$$\mathcal{L}_{\text{Higgs}} = \left(D_\mu H\right)^\dagger \left(D^\mu H\right) - \lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2 \tag{4}$$

$$\mathcal{L}_{\text{Yukawa}} = -\left(Y^{ij}_u \bar{Q}_{Li} \epsilon H^* u_{Rj} - Y^{ij}_d \bar{Q}_{iL} H d_{Rj} - Y^{ij}_e \bar{L}_{Li} H e_{Rj}\right) + h.c. \tag{5}$$

where are 3×3 Yukawa matrices of Yukawa couplings, fields $\bar{Q}_L$ and $\bar{L}_l$ are left-handed doublet fields for quarks and leptons, and u_R, d_R and e_R are singlet fields for up-type quark, down-type quark and lepton. The gauge covariant derivative of Eq. (4) is given in this form:

$$D_\mu = \partial_\mu + i g_3 G^A_\mu T^A + i g_2 W^a_\mu T^a + i g_1 B_\mu Y \tag{6}$$

with g_3, g_2 and g_1 as gauge coupling constants of $SU(3), SU(2)$ and $U(1)$, and T^A, T^a and Y are the $SU(3), SU(2)$ and $U(1)$ generators, respectively.

3. GEORGI-GLASHOW $SU(5)$

Late 1960s and 1970s were the years of blooming period for GUT (Grand Unified Theories). One of those theories was a Georgi-Glashow model based on $SU(5)$ group, published in 1974 [1] stating that this particular $SU(5)$ group was a gauge group of the world. The idea was to provide a unification of all known states from Standard Model on some greater level such as $SU(5)$ group.

3.1. Particle Content

As all the states are in $SU(5)$ group, this would mean that the ground state of the group must be 5-dimensional fundamental representation of $SU(5)$ group:

$$\psi_\alpha = \bar{5} = \begin{pmatrix} d_{R1}^C \\ d_{R2}^C \\ d_{R3}^C \\ e_L \\ -\nu_{eL} \end{pmatrix} = (\bar{3}, 1, 1/3) \oplus \left(1, 2, \frac{1}{2}\right) \quad (7)$$

Since in the Standard Model there are 15 states (Weyl fermions), there were 10 more (beside five states in Eq. (7)) missing. Those ten were provided in the following matter:

$$5 \otimes 5 = 10 \oplus 15 \quad (8)$$

This way, the missing ten states reside in 10-dimensional representation:

$$10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^C & -u_2^C & u_1 & d_1 \\ -u_3^C & 0 & u_1^C & u_2 & d_2 \\ u_2^C & -u_1^C & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^C \\ -d_1 & -d_2 & -d_3 & -e^C & 0 \end{pmatrix} \quad (9)$$

Now the particle content can be given in the Table 2.

Table 2. $SU(5)$ particle content

States	$SU(3)$ irrep	$SU(2)$ irrep	$U(1)$ Hypercharge Y	Charge Q	Weak Isospin T_3	Color
d_{Ri}^C	$\bar{3}$	1	$+\frac{1}{3}$	$+\frac{1}{3}$	0	$i = 1,2,3$
u_{Ri}^C	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{3}$	0	$i = 1,2,3$
d_{Li}	3	2	$+\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$i = 1,2,3$
u_{Li}	3	2	$+\frac{1}{6}$	$+\frac{2}{3}$	$+\frac{1}{2}$	$i = 1,2,3$
e_R^C	1	1	+1	+1	0	-
e_L	1	2	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	-
ν_{eL}	1	2	$-\frac{1}{2}$	0	$+\frac{1}{2}$	-

3.2. Adjoint representation and symmetry breaking

Gauge groups or gauge symmetries for GUTs are at some point or at one or more stages broken in order to go from energy scale of unification to the energy scale of our current achievable energy scale in experiments. Adjoint representation of $SU(5)$ is 24-dimensional representation with 24 generators. This representation is responsible for symmetry breaking that in this gauge group happens at the levels: firstly breaking symmetry of $SU(5)$ to the symmetry group of Standard Model, and secondly breaking symmetry of Standard Model to $SU(3) \times U(1)_{em}$. In first stage there are twelve generators corresponding to gauge bosons leading to proton decay, and the remaining twelve correspond to the Standard Model generators – eight from $SU(3)$, three from $SU(2)$ and one from $U(1)$. The gauge boson of 24 adjoint representation takes the following form:

$$V_\mu = \begin{pmatrix} G_1^1 - \frac{2B}{\sqrt{30}} & G_2^1 & G_3^1 & X_1 & Y_1 \\ G_1^2 & G_2^2 - \frac{2B}{\sqrt{30}} & G_3^2 & X_2 & Y_2 \\ G_1^3 & G_2^3 & G_3^3 - \frac{2B}{\sqrt{30}} & X_3 & Y_3 \\ \bar{X}_1 & \bar{X}_2 & \bar{X}_3 & \frac{W^0}{\sqrt{2}} + \frac{3B}{\sqrt{30}} & W^+ \\ \bar{Y}_1 & \bar{Y}_2 & \bar{Y}_3 & W^- & -\frac{W^0}{\sqrt{2}} + \frac{3B}{\sqrt{30}} \end{pmatrix} \quad (10)$$

where matrix entries represent:

$$G_2^1 = \frac{(g_1 - ig_2)}{\sqrt{2}} \qquad G_1^2 = \frac{(g_1 + ig_2)}{\sqrt{2}}$$
$$G_3^1 = \frac{(g_4 - ig_5)}{\sqrt{2}} \qquad G_1^3 = \frac{(g_4 + ig_5)}{\sqrt{2}}$$
$$G_3^2 = \frac{(g_6 - ig_7)}{\sqrt{2}} \qquad G_2^3 = \frac{(g_6 + ig_7)}{\sqrt{2}}$$
$$G_1^1 = \frac{g_3}{\sqrt{2}} + \frac{g_8}{\sqrt{6}} \qquad G_2^2 = -\frac{g_3}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}$$
$$G_3^3 = -g_8 \quad (11)$$

with g_i $(i = 1, \dots, 8)$ being gluons and X and Y gauge bosons leading to proton decay. As we mentioned earlier, symmetry breaking [7] is occurring at two stages – first with 24_H from $SU(5)$ to Standard Model where gauge bosons X and Y acquire mass, and second stage with 5_H from Standard Model to electroweak symmetry where Standard Model states acquire their masses.

3.3. Problems with Georgi-Glashow model

Georgi-Glashow mass generating Lagrangian can be represented in the following form:

$$\mathcal{L}_{\text{Yukawa}} = Y_u \varepsilon_{ijklm} \bar{\psi}_{10}^{C\,ij} \psi_{10}^{kl} 5_H^m + Y_d \bar{\psi}_{5a} \psi_{10}^{ab} 5_{Hb}^* + \text{h.c.} \quad (12)$$

At the second stage of symmetry breaking, the simplest Higgs can be used and transformed as the fundamental representation of $SU(5)$. Substituting it in Eq. (12), it will give:

$$\mathcal{L} = -\frac{Y_d v}{\sqrt{2}} \left[\bar{d}_R d_L + \bar{e}_R^C e_L^C + \text{h.c.} \right] \quad (13)$$

where must be performed summation over index of color and index of flavor. This is what leads to degeneracy in mass for charged leptons (electron, muon and tau) and down-type quarks (d-quark – down, s-quark – strange, and b-quark – bottom). One can conclude that Y_d is common Yukawa coupling matrix [1, 8] for both down-type quarks and charged leptons, which gives as a result:

$$m_d = m_e \qquad m_s = m_\mu \qquad m_b = m_\tau \quad (14)$$

One of another problem with this model is that neutrinos are massless as in the Standard Model as well. However, experimental results demonstrate that neutrinos are not massless particles, at least not all of them. Therefore, Georgi-Glashow offers a great idea for unification, but with some extensions in order to solve this disagreement between theory and experiment.

4. EXTENSIONS OF GEORGI-GLASHOW MODEL

Georgi-Glashow model has been an inspiration and a basis for various modifications and extensions through history for different explorations and fields of research, from unification theories and proton decay [9-12], to magnetic monopoles [13]. One of the most prominent and simplistic extensions at the same time is the new model [9-12] where fermion sector is extended with $15_F + \overline{15}_F$ representations and scalar sector is extended with 35_H representation.

4.1. Neutrino mass generating mechanism

These new extensions in fermion and scalar sector comprise overall seven new physics states - Σ_1, Σ_3 and Σ_3 for $15_F + \overline{15}_F$ and Φ_1, Φ_3, Φ_6 and Φ_{10} for 35_H. Among these seven states, two are responsible for generating neutrino masses - Σ_1 and Φ_1.

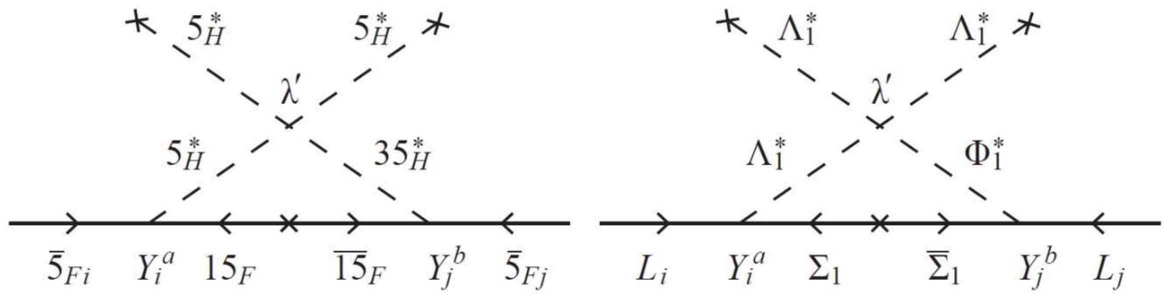

Figure 1. The Feynman diagrams of the leading order contribution towards Majorana neutrino masses at the $SU(5)$ (left panel) and the Standard Model (right panel) levels [9-12]

This mechanism generates two out of three neutrino masses, whilst one neutrino remains massless. Experiments [14-16] provide results for differences between squares of neutrinos masses, therefore not suggesting that all three must have mass. Within this extension, neutrino ordering is of normal hierarchy.

4.2. Proton decay

Proton decay can be studied as a problem of its own or as a consequence of a unification, since it takes high energy mediating particles for process to occur. In non-supersymmetric models, this happens via dim-6 operators. Namely, it decays via gauge bosons whose mass is of mass of unification. However, in extended models, such as [9-12], there is another mediating particle – scalar leptoquark. This new mediating particle has a lower mass limit than gauge bosons. Proton decay can be represented via Feynman diagrams – Fig. 2 shows proton decay via gauge boson mediation where one quark is spectator quark, and other two are participating in decay in initial state. In the final state, products are lepton (positron or anti-muon or anti-neutrino) and anti-quark that in combination with spectator quark forms a meson. In Fig. 3 is represented proton decay via scalar leptoquark. Again, in final states there are two products – lepton and meson. Overall, there are eight channels for proton to decay:

$$p \to \pi^0 e^+, p \to \pi^0 \mu^+, p \to \eta^0 e^+, p \to \eta^0 \mu^+, p \to K^0 e^+, p \to K^0 \mu^+, p \to \pi^+ \bar{\nu}, \text{and } p \to K^+ \bar{\nu}. \quad (15)$$

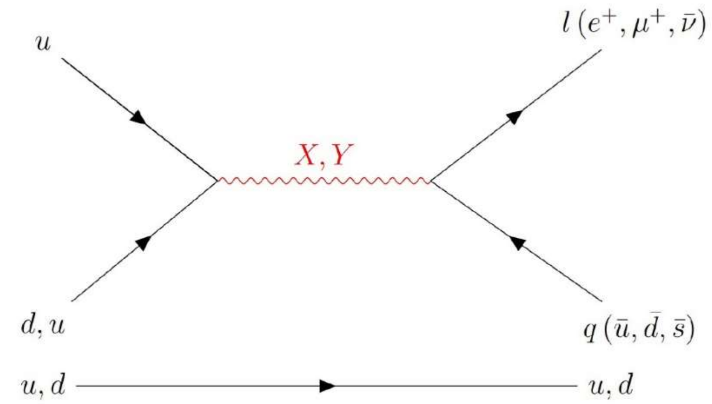

Figure 2. Proton decay via the gauge boson mediation

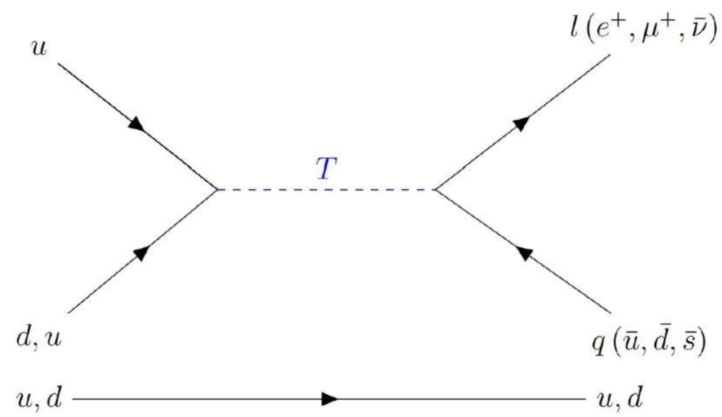

Figure 3. Proton decay via the scalar leptoquark mediation

5. CONCLUSION

Georgi-Glashow $SU(5)$ was one of the simplest models to try to achieve unification at greater energy scale and resolve problems that Standard Model encountered. However, this simplistic $SU(5)$ model could not have explain neutrino masses and therefore leaving neutrinos massless as in Standard Model. In addition, it showed degeneracy in mass sector for charged leptons and down-type quarks.

Nevertheless, it seemed that it was almost there to solve the unification problem, neutrinos masses and proton decay. But something was missing. That something was searched for in various models, with some not so simplistic, and not tackling all the issues Georgi-Glashow model came across. The model that solves all those problems was introduced in recent years [9-12]. This is minimalistic non-supersymmetric model with minimal extension of Georgi-Glashow model. Therefore, Georgi-Glashow model, despite its weaknesses or defects, was pioneering model giving a strong basis for today's models successfully solving problems initial and original $SU(5)$ had, with extensions of new physics' states.

Conflicts of Interest

The authors declare no conflict of interest.

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